Maxwell's Electrodynamics Without Special Relativity Theory (Part I)

Victor N. Barykin
Scientific Center 'Malinovka'
15-234 Str. Slobodskaya, 220098 Minsk, BELARUS

This work suggests for Maxwell's electrodynamics in moving media a generalization that 1) does not resort to Einstein's special relativity theory, 2) bases its calculations and experiments on Newton's space, 3) naturally incorporates superluminal velocities and indicates the requirements for the latter to be discovered, and 4) describes in a unified manner the classical experiments of Bradley, Fizeau, Michelson, and Doppler.

Introduction

The present work shows the possibility of dynamic description for a change of the inertial properties of an electromagnetic field, when its frame of reference is considered as a physical environment capable of influencing the parameters of the field. This description is conceived within the framework of the Newtonian space-time, and articulated within a single coordinate system.

Maxwell's Dynamic Equations in Newtonian Space-Time

We will start with the concept of a single observer who has standards of length and time according to Newton's space-time model $R^3 \times T^1$. The physical laws of Maxwell's electrodynamics in $R^3 \times T^1$ can be determined in terms of the three-dimensional operators $\nabla\times$ and $\nabla\cdot$, and they have the vector form:

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},\ \nabla\cdot\mathbf{B} = 0$$

$$\nabla\cdot\mathbf{D} = 4\pi\rho,\ \nabla\times\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} + 4\pi\frac{\mathbf{J}}{c}$$

In the algebra $F(4)$, field elements form

$$F_{mn} = \begin{pmatrix} 0 & B_z & -B_y & -iE_x \\ -B_z & 0 & B_x & -iE_y \\ B_y & -B_x & 0 & -iE_z \\ iE_x & iE_y & iE_z & 0 \end{pmatrix}$$

$$H^{ik} = \begin{pmatrix} 0 & H_z & -H_y & -iD_x \\ -H_z & 0 & H_x & -iD_y \\ H_y & -H_x & 0 & -iD_z \\ iD_x & iD_y & iD_z & 0 \end{pmatrix}$$

Maxwell's equations acquire the tensor form $\partial_{[k}F_{mn]} = 0$, $\partial_k H^{ik} = S^i$, where ∂_k is the covector of partial derivatives; for example, over the coordinates $x^1 = x$, $x^2 = y$, $x^3 = z$, $x^0 = ict$.

Physically speaking, these sets of equations are equivalent; however, it is more convenient to carry out the mathematical analysis of general problems in tensor form. Starting from these equations, following the model of dynamic change of field parameters in Newtonian space-time, and not resorting to the concept of an ether, we will describe in a unified manner the experiments of Bradley [1], Fizeau [2], Michelson [3], and Doppler [4], plus the 'constancy' of light speed in vacuum [5].

Generalized Connections Between Fields and Inductions in Maxwell's Electrodynamics

For an isotropic medium at rest, the connection between fields and inductions has the form: $\mathbf{D} = \varepsilon\mathbf{E}$, $\mathbf{B} = \mu\mathbf{H}$, where ε and μ are the dielectric and magnetic permeabilities. In the version considered by Minkowski [6], the medium is a secondary radiation source, so the medium velocity $\mathbf{U}_m$ is identical with this velocity of the radiation source:

$$\mathbf{D} + \left[(\mathbf{U}_m / c)\times\mathbf{H}\right] = \varepsilon\left(\mathbf{E} + \left[(\mathbf{U}_m / c)\times\mathbf{B}\right]\right)$$

$$\mathbf{B} + (\mathbf{E}\times\mathbf{U}_m / c) = \mu\left[\mathbf{H} + (\mathbf{D}\times\mathbf{U}_m / c)\right]$$

We will seek new connections between the fields F_{mn} and inductions H^{ik} [7] in the form $H^{ik} = \Omega^{im}\Omega^{kn}F_{mn}$. Let Ω^{im} be equal to $\Omega^{im} = \alpha(\Theta^{im} + \beta U^i U^m)$, where α and β are scalar functions, $\Theta^{im} = \mathrm{diag}(1, 1, 1, \chi)$ is the metric tensor in $R^3\times T^1$, and $\chi = \det\Theta^{im}$, $U^i = dx^i / d\Theta$ represents the four-velocities, but without invoking SRT.

We have $d\Theta^2 = \Theta_{ij}dx^i dx^j$, and the inverse tensor Θ_{ij} can be specified in two ways: **a)** $\Theta_{ij}\Theta^{jk} = \delta_i^k$, or **b)** $\Theta_{ij} = b_{ik}b_{jl}\Theta^{kl}$, where b_{ij} are additional tensors. For any such statement, the expression for Ω^{im} has been found in [8] by solving a system of nonlinear algebraic equations following from the generalized formal connection between fields and inductions, when the connections are considered for zero velocity. Then

$$\Omega^{im} = \frac{1}{\sqrt{\mu}}\left[\Theta^{im} + \left(\frac{\varepsilon\mu}{\chi} - 1\right)U^i U^m\right]$$

The tensor Ω^{im} has no singularity at $\chi = 0$. Actually,

$$d\Theta = \frac{icdt}{\sqrt{\chi}}\sqrt{1-\chi U^2/c^2}, \quad U^k = \frac{dx^k}{d\Theta} = \frac{\sqrt{\chi}}{ic}\frac{dx^k}{dt}\Big/\sqrt{1-\chi U^2/c^2}$$

For the velocities $U_n = \Theta_{nk}U^k$ we have $U^kU_k = 1$. In view of the antisymmetry of F_{mn} and H^{ik}, we have

$$H^{ik} = \Omega^{ikmn}F_{mn}, \quad \Omega^{ikmn} = 0.5(\Omega^{im}\Omega^{kn} - \Omega^{in}\Omega^{km})$$

with the conditions $\Omega^{ikmn} = -\Omega^{iknm} = -\Omega^{kimn}$.

Maxwell's generalized equations take the vector form [9]:

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \quad \nabla\cdot\mathbf{B} = 0, \quad \nabla\cdot\mathbf{D} = 4\pi\rho, \quad \nabla\times\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j}$$

$$\mathbf{D} + \chi[(\mathbf{U}/c)\times\mathbf{H}] = \varepsilon(\mathbf{E} + [(\mathbf{U}/c)\times\mathbf{B}])$$

$$\mathbf{B} + \chi[\mathbf{E}\times\mathbf{U}/c] = \mu(\mathbf{H} + [\mathbf{D}\times\mathbf{U}/c])$$

The Main Modeling Problem

Let a radiation source move around the Earth in vacuum with instantaneous velocity $\mathbf{U}_{fs}$, which is the velocity of the primary radiation source $\mathbf{U}|_{\rho=0} = \mathbf{U}_{fs}$. Let the radiation spread from empty space into the atmosphere of Earth, which has density ρ, in which for $\rho = \rho_0$ the velocity of the secondary radiation source is equal to the velocity of the physical medium $\mathbf{U}_m$:

$$\mathbf{U}|_{\rho=\rho_0} = \mathbf{U}_m$$

Let us introduce the velocity $\mathbf{U} = \mathbf{U}(\mathbf{U}_{fs}, \mathbf{U}_m, w(\rho))$, assuming that it also depends on a functional $w(\rho)$, which is called the phase of the electromagnetic field inertia. In agreement with the physical formulation indicated in [7], we will assume that the velocity $\mathbf{U}$ is governed by the relaxation equation

$$d\mathbf{U}/d\xi = -P_0(\mathbf{U} - \mathbf{U}_m), \quad \mathbf{U}|_{\xi=0} = \mathbf{U}_{fs}$$

Here P_0 is the relaxation constant, $\xi = \rho/\rho_0$. The solution of the relaxation equation is

$$\mathbf{U} = (1-w)\,\mathbf{U}_{fs} + w\,\mathbf{U}_m, \quad w = 1 - \exp(-P_0\rho/\rho_0)$$

We have the conditions

$$\mathbf{U}|_{\rho=0_0} = \mathbf{U}_{fs}, \quad w|_{\rho=0} = 0, \quad \mathbf{U}|_{\rho=\rho_0} = \mathbf{U}_m, \quad w|_{\rho=\rho_0} = 1$$

We require that $\chi = w$. Then the solution of the problem indicated is in general possible.

Solution of Maxwell's Generalized Equations with Constant w

When $w = \text{const}$, the equations for the field potentials A_m in their four-dimensional form are [10]:

$$\left[\Theta^{kn}\frac{\partial}{\partial x^k}\frac{\partial}{\partial x^n} - (\varepsilon\mu - w)\left(U^k\frac{\partial}{\partial x^k}\right)^2\right]A_m = -\mu U^i\Theta_{im}$$

with the calibration condition:

$$\Theta^{kn}\frac{\partial A_n}{\partial x^k} - (\varepsilon\mu - w)\frac{\partial A_l}{\partial x^k}U^lU^k = 0$$

According to the definitions for the vector and scalar potentials $\mathbf{A}$ and φ,

$$\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} - \nabla\varphi, \quad \mathbf{B} = \nabla\times\mathbf{A}$$

we obtain

$$\hat{L}\mathbf{A} = -\frac{4\pi\mu}{c}\left\{\mathbf{J} + \frac{\sigma^2}{\sigma+w}\frac{\mathbf{U}}{c}\left(w\mathbf{U}\cdot\mathbf{J} - c^2\rho\right)\right\}$$

$$\hat{L}\varphi = -4\pi\mu\frac{\Gamma^2}{w+\sigma}\left\{\rho\left(1 - \varepsilon\mu U^2/c^2\right) + \sigma\mathbf{U}\cdot\mathbf{J}/c^2\right\}$$

and the calibration condition

$$\left(\nabla\cdot\mathbf{A} + \frac{w}{c}\frac{\partial^2}{\partial t^2}\right) - \frac{\sigma\Gamma^2}{c^2}\left(\frac{\partial}{\partial t} + \mathbf{U}\cdot\nabla\right)\left(\vec{\mathbf{U}}\cdot\vec{\mathbf{A}} - c\varphi\right) = 0$$

wherein

$$\hat{L} = \left(\Delta - \frac{w}{c^2}\frac{\partial^2}{\partial t^2}\right) - \sigma\frac{\Gamma^2}{c^2}\left(\frac{\partial}{\partial t} + \mathbf{U}\cdot\nabla\right)^2$$

$$\sigma = \varepsilon\mu - w, \quad \Gamma^2 = \left(1 - w\beta^2\right)^{-1}, \quad \beta = \frac{U}{c}$$

The Green's function for the vector equations is indicated in [7]:

$$G_0(\mathbf{r},t) = \frac{16\pi^4\mu}{\sqrt{r^2+\xi^2}}\delta\left[t - \frac{1}{c}\frac{\varepsilon\mu - \beta^2w^2}{(1-w\beta^2)\sqrt{\varepsilon\mu}}\sqrt{r^2+\xi^2}\right]$$

In a cylindrical coordinate system, the position vector has length $R = (\rho^2 + z^2)^{1/2}$, and the parameter values are

$$r^2 = \rho^2\frac{\varepsilon\mu(1-w\beta^2)}{\varepsilon\mu - \beta^2w^2}, \quad \xi = z - \frac{\varepsilon\mu - w}{\varepsilon\mu - \beta^2w^2}Ut$$

When $\beta = 0$, we have the Green's function for the medium at rest without dispersion:

$$G_0(\mathbf{r}, t)|_{\mathbf{U}=\mathbf{0}} = 16\pi^4\mu\frac{1}{R}\sigma\left(t - R\sqrt{\varepsilon\mu}/c\right)$$

The Green's function differs from zero on the surface

$$t=\frac{1}{c}\frac{\varepsilon\mu-\beta^2w^2}{(1-w\beta^2)\sqrt{\varepsilon\mu}}\sqrt{\rho^2\frac{\varepsilon\mu(1-w\beta^2)}{\varepsilon\mu-\beta^2w^2}+\left(z-\frac{\varepsilon\mu-w}{\varepsilon\mu-\beta^2w^2}Ut\right)^2}$$

This is an ellipsoid of rotation whose symmetry axis coincides with $\bar{U}$, and the position of the center is given by

$$z_0=Ut\frac{\varepsilon\mu-w}{\varepsilon\mu-\beta^2w^2}$$

The center of this ellipsoid moves with the velocity

$$U_0=U\frac{\varepsilon\mu-w}{\varepsilon\mu-\beta^2w^2}$$

The semiaxes of the ellipsoid are equal to

$$a=ct\sqrt{\frac{1-w\beta^2}{\varepsilon\mu-\beta^2w^2}}\ ,\ b=ct\frac{\sqrt{\varepsilon\mu}\,(1-w\beta^2)}{\varepsilon\mu-\beta^2w^2}$$

Recalling operator $\hat{L}$, the dispersion equation for the electromagnetic field has standard form [11]:

$$c^2K^2=w\omega^2+\Gamma^2(\varepsilon\mu-w)(\omega-\mathbf{K}\cdot\mathbf{U})^2,\ \ \Gamma^2=1/(1-w\beta^2)$$

where $\mathbf{K}$ is the wave vector. This yields the expression for the group velocity:

$$\mathbf{V}_g=\frac{\partial\omega}{\partial\mathbf{K}}=c\frac{\mathbf{K}+\sigma\Gamma^2c^{-2}U(\omega-\mathbf{K}\cdot\mathbf{U})}{w\omega/c+\sigma\Gamma^2c^{-1}(\omega-\mathbf{K}\cdot\mathbf{U})}$$

In the nonrelativistic limit,

$$\mathbf{V}_g=\frac{c}{n}\frac{\mathbf{K}}{K}+(1-w/n^2)\left[(1-w)\,\mathbf{U}_{fs}+w\mathbf{U}_m\right]$$

Analysis of the Expressions Obtained

1. At $w=0$ we have $\mathbf{V}_g=c\mathbf{K}/K+\mathbf{U}_{fs}$. Thus, in the generalized model of electromagnetic events the field moves such that the center of the surface on which the Green function is nonzero moves with the velocity $\mathbf{U}_{fs}$, and the semiaxes of the ellipsoid in this case are equal, giving a sphere. This picture corresponds to an intuitive comprehension of the fact that, according [12], in the absence of external influences, the field in vacuum retains its inertia throughout its propagation external to its primary source.

2. The generalized electrodynamics of Maxwell's one is consistent with the experiments of Michelson [3]. According to the conditions of his experiment, the velocity of the medium was equal to zero, $\mathbf{U}_m=\mathbf{0}$, just as the velocity of the radiation source. For this reason we have the radiation velocity to be independent of the direction:

$$\mathbf{V}_g=\frac{c}{n}\frac{\mathbf{K}}{K}$$

3. The generalized electrodynamics of Maxwell is consistent with the experiment of Fizeau [2]. According to the experimental conditions $\mathbf{U}_{fs}=\mathbf{0}$ and $w=1$, therefore the velocity is equal to

$$\mathbf{V}_g=\frac{c}{n}\frac{\mathbf{K}}{K}+\left(1-\frac{1}{n^2}\right)\mathbf{U}_m$$

Conclusions

The generalization of Maxwell's electrodynamics, which allows one to describe in a unified manner a vast quantity of experimental data without resorting to special relativity theory, is possible if the connections between the fields and inductions is taken into account.

The functional $w(\rho)$ and also the velocity that specifies the external inertia of the field $\mathbf{U}=\left(1-w\right)\mathbf{U}_{fs}+w\mathbf{U}_m$, which are the determining factors for the velocity $\mathbf{V}_g$ and the frequency ω of the electromagnetic field, change dynamically in this case.

References

[1] J. Bradley, "A new apparent motion discovered in the fixed stars; its cause assigned; the velocity and aquable motion of light deduced" Phil. Trans 35, 637-653 (1728).

[2] H. Fizeau, "Sur les hypothéses relatives a l'éther lumineux et sur un experiment qui parait démontrer que mouvement des corps change la vitesse; avec laquelle la lumiére se propage dans leur interieur", Comp. Rend. 33, 349-355 (1851).

[3] A. Michelson, "The relative motion of the Earth and luminiterous aether", Amer. J. Phys. 22, 120-129 (1881).

[4] Ch. Doppler, " Über das farbige Light der Doppelsterne und einiger andern Gestirne des Himmels", ABH. Böhm. Ges, b. 2, s. 465 (1842).

[5] U.I. Frankfurt, "Optics of moving medium and SRT", Einst. mat. 1977, pp. 257-325 (in Russian, M.: Science, 1980)

[6] H. Minkowski, "A deduction of the basic equations for electromagnetic processes in moving bodies from the point of view of the electron theory", Einst. mat. 1978-79, pp. 64-91 (in Russian, M.: Science, 1983).

[7] V.N. Barykin, "Towards mathematical description of electromagnetic phenomena in a moving rarefied gas", Izv. VUZov, Fizika 10, 54-58 (1990) (in Russian).

[8] V.N. Barykin, "New time-space symmetries in the electrodynamics of media", Izv. VUZov, Fizika 10, 26-30 (1986) (in Russian).

[9] V.N. Barykin, "About the physical complementarity of the Galilea and Lorenz groups in the electrodynamics of isotropic inertially moving media", Izv. VUZov, Fizika 9, 57-66 (1989) (in Russian).

[10] V.N. Barykin, "Time-space symmetries in the electrodynamics of isotropic inertially moving media", Group-theoretic methods in physics, pp. 461-466 (in Russian, M.: Science, vol. 1, 1986).

[11] S.N. Stolyarov, "The boundary problems of the electrodynamics moving media", Einst. ma't. 1975-1976, pp. 152-215 (in Russian, M.: Science, 1978).

[12] W. Ritz, "Rechercher critique sur l' electrodynamique générale", Ann. Chim 13 (8) 145-275 (1908).