Mathematical basis of the electromagnetic radiation structure

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Abstract

The generalized Maxwell's equations for the moving media, allowing one to describe dynamically the relativistic effects without the special relativity theory, were analyzed. They were written in the form of the G modulus for the PSL(4,R) group in the monomial representation. These equations involve two metrics: the Minkowski metric and the Euclidean one. The PSL(4,R) group was interpreted as a system of relations between the four abstract objects. A structural model of the light particles in the form of objects consisting of a pair of electric precharges and a pair of gravitational precharges is proposed. A formula for the Planck constant and the light particle energy has been derived.

Keywords: light particle energy, mechanical model of the light, generalization of Maxwells electrodynamics

Introduction

The possibility of consideration of an electromagnetic radiation as a system of dimensional particles in the physical three-dimensional space was mathematically substantiated in this article.

In the special relativity theory there is no any proposition that light has a physical structure. Within the framework of this theory, it is impossible to formally consider, with no logical contradictions, a light quantum of finite size in an rest frame of reference. Because of this, light cannot have a combined structure in the usual physical sense.

Therefore, to analyze the light structure, it is necessary to develop a generalized model of electromagnetic phenomena. On the one hand, this model should be, in its consequences and properties, beyond the scope of the Einstein symmetry approach. On the other hand new model should give a way and means for positive construction of a structural model of light. If this step is possible the consideration of radiation with the use of the structural approach is not contradictory to the special relativity theory.

The description of radiation with the use of the structural approach is not in conflict with the quantum electrodynamics that has replaced the classical electrodynamics because of the necessity to take into account the discrete properties of radiation. The quantum electrodynamics has proved its efficiency in the description of the majority of experimental data without recourse to the notion that light has a combined structure. It has been established that light can be described to advantage to the nuclear scale: the lengths of the order of the nucleon size, with the use of the structureless point approach to the light. However, light can has a more "thin", subnuclear structure. This possibility does not negates of the quantum electrodynamics and does not disproves it.

The viewpoint of experimenters that light is a material substance differs from the viewpoint of theorists on light. Beginning in 1960, an enormous number of experiments on the light structure, including modern large reviews [1]-[7], have been carried out. It has been established experimentally that the interaction of photons and hadrons is similar to the interaction of hadrons. It is known that hadrons have structure. We can propose that the light have some physical structure with the analogy of hadrons.

By now more than 20 composite models of light have been developed. However, at present there is no a unique view on the base objects, of which light consists, and there is no a mechanical model of motion in a light particle, which would be consistent with the experimental data.

In the present work, the possibility of construction of a composite mechanical model of the light particle was investigated. We shall start from the idea that the Maxwell equations written in the matrix form "prompt" the structure of a light particle. The base object for the light particle consist of a pair of electrically and gravitationally neutral objects moving relative to each other. As a generalized model of electromagnetic phenomena, the electrodynamics with the index of relation [8] was used. It was developed in 1985 for dynamic description of relativistic effects. This model generalizes the special relativity theory, however, it is free of its limitations, and, therefore, opens the way for construction of a structural model of light with physical dimensions in the three-dimensional space.

The usefulness of the approach proposed is warranted by the fact that it allows one to derive an equation for the energy of a light particle and, particularly, an equation for the Planck constant.

1. Constructibility of the generalized Maxwell equations in the matrix form

We shall use the generalized electrodynamics model that is free of the limitations of the special relativity theory, generalizes this theory, and dynamically represents relativistic effects. In this model, the dynamics of fields $\left(\vec{E},\ \vec{B}\right)$ and inductions $\left(\vec{H},\ \vec{D}\right)$ is determined by the Maxwell equations

$$\nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t},\ \nabla\cdot\vec{B}=0,$$

$$\nabla\cdot\vec{D}=4\pi\rho,\ \nabla\times\vec{H}=\frac{1}{c}\frac{\partial\vec{D}}{\partial t}+4\pi\frac{\vec{J}}{c}.$$

The fields and inductions are related by the equations

$$\vec{D}+w\left[\frac{\vec{U}}{c}\times\vec{H}\right]=\varepsilon\ \left(\vec{E}+\left[\frac{\vec{U}}{c}\times\vec{B}\right]\right),$$

$$\vec{B}+w\left[\vec{E}\times\frac{\vec{U}}{c}\right]=\mu\ \left(\vec{H}+\left[\vec{D}\times\frac{\vec{U}}{c}\right]\right),$$

where ε,μ — are the permittivity and permeability respectively, U_x,U_y,U_z — are the composite velocity components, c — is the velocity of light in the vacuum. The model being considered includes the quantities

$$w=1-\exp\left[-P_0\ (n-1)\right],\ \vec{U}=(1-w)\ \vec{U}_{fs}+w\,\vec{U}_m,$$

where $\vec{U}_{fs}$ is the velocity of the primary radiation source, $\vec{U}_m$ is the velocity of the medium, *w is the index of relation* (a new scalar quantity introduced into the electrodynamics), n is the index of refraction, $P_0\left(\lambda\right)$ is an empirical quantity dependent on the wavelength radiation.

From the above-described model, new laws for light follow. For example, the group velocity of an electromagnetic field depends, in the nonrelativistic limit, not only on the index of the refraction, but also on the index of relation as well as not only on the velocity of the medium, but also on the velocity of the primary radiation source. In the nonrelativistic limit we have the formula

$$\vec{V}_g=\frac{c}{n}\frac{\vec{K}}{K}+\left(1-\frac{w}{n^2}\right)\left[(1-w)\ \vec{U}_{fs}+w\vec{U}_m\right].$$

Proposed model and its consequences are described in detail in [8]-[10].

Let us represent the generalized electrodynamics equations in the matrix form in the coordinates $x^1=x$, $x^2=y$, $x^3=z$, $x^0=ict$. We shall use two contravariant tensors $g^{kn}=diag\left(1,1,1,-1\right), r^{kn}=diag\left(1,1,1,1\right)$, the quantities

$$\Psi=\begin{pmatrix}E_x+iB_x\\E_y+iB_y\\E_z+iB_z\\0\end{pmatrix},\ \Psi^*=\begin{pmatrix}E_x-iB_x\\E_y-iB_y\\E_z-iB_z\\0\end{pmatrix},$$

$$\varphi=\begin{pmatrix}H_x+iD_x\\H_y+iD_y\\H_z+iD_z\\0\end{pmatrix},\varphi^*=\begin{pmatrix}H_x-iD_x\\H_y-iD_y\\H_z-iD_z\\0\end{pmatrix}$$

and two groups $a_1,a_2,a_3,a_0\in A$, $b_1,b_2,b_3,b_0\in B$:

$$a_1=\begin{pmatrix}0&-\sigma_2\\-\sigma_2&0\end{pmatrix},\ a_2=\begin{pmatrix}0&-\sigma_0\\\sigma_0&0\end{pmatrix},$$

$$a_3=\begin{pmatrix}\sigma_2&0\\0&-\sigma_2\end{pmatrix},\ a_0=\begin{pmatrix}\sigma_0&0\\0&\sigma_0\end{pmatrix},$$

$$b_1=\begin{pmatrix}0&\sigma_1\\-\sigma_0&0\end{pmatrix},\ b_2=\begin{pmatrix}0&-\sigma_3\\\sigma_3&0\end{pmatrix},$$

$$b_3=\begin{pmatrix}\sigma_2&0\\0&\sigma_2\end{pmatrix},\ b_0=\begin{pmatrix}\sigma_0&0\\0&\sigma_0\end{pmatrix},$$

where

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

We use so the following matrices:

$$A_1 = \begin{pmatrix} 0 & a(1) \\ a(3) & 0 \end{pmatrix}, \ A_2 = \begin{pmatrix} 0 & a(2) \\ a(4) & 0 \end{pmatrix},$$
$$A_3 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & a(5) \end{pmatrix}, \ A_0 = a_0,$$
$$B_1 = \begin{pmatrix} 0 & b(1) \\ b(3) & 0 \end{pmatrix}, \ B_2 = \begin{pmatrix} 0 & b(2) \\ b(4) & 0 \end{pmatrix},$$
$$B_3 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & b(5) \end{pmatrix}, \ B_0 = b_0,$$

where

$$a(1) = \begin{pmatrix} 0 & -w \\ 1 & 0 \end{pmatrix}, \ a(2) = \begin{pmatrix} -1 & 0 \\ 0 & -w \end{pmatrix},$$
$$a(3) = \begin{pmatrix} 0 & -1 \\ w^{-1} & 0 \end{pmatrix}, \ a(4) = \begin{pmatrix} 1 & 0 \\ 0 & w^{-1} \end{pmatrix},$$
$$a(5) = \begin{pmatrix} 0 & -w \\ w^{-1} & 0 \end{pmatrix}, \ b(1) = \begin{pmatrix} 0 & w \\ 1 & 0 \end{pmatrix},$$
$$b(2) = \begin{pmatrix} -1 & 0 \\ 0 & w \end{pmatrix}, \ b(3) = \begin{pmatrix} 0 & -1 \\ -w^{-1} & 0 \end{pmatrix},$$
$$b(4) = \begin{pmatrix} 1 & 0 \\ 0 & -w^{-1} \end{pmatrix}, \ b(5) = a(5).$$

The elements of the groups are determined accurate to the multiplication by the minus unity. We shall determine the projective operator $P = column\,(1,1,1,1)$. We may write the differential equations in the matrix form, involving pairs of fourmetrics:

$$\left(g^{kn} a_k\, \partial_n\, \Psi^* + r^{kn} b_k\, \partial_n\, \Psi\right) P = 0,$$

$$\left(r^{kn} a_k \partial_n\, \varphi^* + g^{kn} b_k\, \partial_n\, \varphi\right) P = \Phi,$$

here,

$$\Phi = \text{column}\,(2\,\rho U_x,\ 2\,\rho U_y,\ 2\,\rho U_z,\ -2\,i\,\rho)\,.$$

The Faraday–Ampere equations written accurate to the projective operator in the explicit form will be used:

$$\left(a_1\partial_x + a_2\partial_y + a_3\partial_z + a_0\frac{(-i)}{c}\partial_t\right)\Psi^* +$$

$$\left(a_1\partial_x + a_2\partial_y + a_3\partial_z + a_0\frac{(-i)}{c}\partial_t\right)\Psi = 0$$

We can write the relations between the fields and inductions in the analogous matrix form:

$$i\mu\left(a_1\frac{U_x}{c} + a_2\frac{U_y}{c} + a_3\frac{U_z}{c} - ia_0\right)\varphi^*$$
$$- i\mu\left(b_1\frac{U_x}{c} + b_2\frac{U_y}{c} + b_3\frac{U_z}{c} + ib_0\right)\varphi$$
$$= w\left(A_1\frac{U_x}{c} + A_2\frac{U_y}{c} + A_3\frac{U_z}{c} + \frac{i}{w}a_0\right)\Psi^*$$
$$+ w\left(B_1\frac{U_x}{c} + B_2\frac{U_y}{c} + B_3\frac{U_z}{c} - \frac{i}{w}b_0\right)\Psi.$$

The expressions

$$\Psi_1^* = E_x - iB_x = -\frac{1}{c}\frac{\partial A_x}{\partial t} - \frac{\partial \varphi}{\partial x} - i\frac{\partial A_z}{\partial y} + i\frac{\partial A_y}{\partial z}, ...$$
$$\Psi_1 = E_x + iB_x = -\frac{1}{c}\frac{\partial A_x}{\partial t} - \frac{\partial \varphi}{\partial x} + i\frac{\partial A_z}{\partial y} - i\frac{\partial A_y}{\partial z}, ...,$$

have the following matrix representation:

$$(a_1\partial_1 + a_2\partial_2 + a_3\partial_3 - E\partial_\tau)\,A = \Psi^*,$$
$$(b_1\partial_1 + b_2\partial_2 + b_3\partial_3 + E\partial_\tau)\,A = -\Psi,$$

where $A = column(A_x, A_y, A_z, -i\varphi)$. From the mathematical standpoint, the representation of the generalized Maxwell equations in the matrix form is simple and natural. A radically new moment is only the representation of these equations with a pair of the groups A and B.

From the physical standpoint, the matrix form of the Maxwell equations is constructive for the creation of the light structure model. We shall demonstrate this idea. Let us assume that the matrices used in the electrodynamics equations indirectly testify to the structure of the electromagnetic field.

To theoretically substantiate this supposition, we shall consider several experimental facts:

- light has no mass and an electric charge,
- the interaction of $\gamma-$ quanta gives elementary particles with an electric charge and a mass.

Let us take the hypothesis that light is a system of neutral objects contains of positive and negative electrical and gravitational precharges connected with each other using a system of the force lines.

It should be noted that we can not simulate precharges, even though this problem is natural for a real model. We have no information on the possible structure of the force lines connecting the precharges and the laws by which the precharges interact with each other.

2. Filling group for a physical models

It should be noted, that the product of the elements of the groups A and B gives new elements. The totality of them is the projective unimodular group $PSL\,(4, R)$ determined by the monomial matrices:

$$e_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix},\ e_3 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix},$$

$$b_3 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix},\ c_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix},$$

$$e_2 = \begin{pmatrix} 0 & \sigma_0 \\ \sigma_0 & 0 \end{pmatrix},\ e_1 = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix},$$

$$a_1 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix},\ f_2 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix},$$

$$b_1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix},\ f_1 = \begin{pmatrix} 0 & -\sigma_2 \\ \sigma_2 & 0 \end{pmatrix},$$

$$b_2 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix},\ c_2 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix},$$

$$f_3 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix},\ a_3 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix},$$

$$c_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix},\ a_2 = \begin{pmatrix} 0 & \sigma_0 \\ -\sigma_0 & 0 \end{pmatrix}.$$

This group can be interpreted as a set of mutual relations in the system consisting of four objects. Restricting ourselves only to the canonical relations, we shall represent them in the first row for the first object, in the second row for the second object, and so on. To a column corresponds the ordinal number of an object being analyzed. Let the ratios of the canonical system of numbers will be $[-1, 0, 1]$. As a result, we shall obtain a system of monomial matrices. The above-indicated variant, determining a group, corresponds to a "sample' from the totality. We shall call it the group of filling of a physical model.

The indicated group is sufficient to represent the matrix algebra elements of dimension 4×4 as a linear superposition. Since the fundamental physical models admit a matrix representation, the group PSL(4,R) warrants its name. Note that the filling group confirmes, from the mathematical point jf view, the physical idea that light is a physical system, consisting of four objects (precharges) with the mutual relations.

The elements of the group are divided into the subgroups a_i, b_i, c_i, e_i, f_i . The matrices a_i, b_i determine a pair of quaternions; they commutate and, as a result of the mutual multiplications, give birth to the other elements of the group. The matrices e_i, f_i determine a pair of antiquaternions —- they anticommutate. The matrices c_i "transform" a_i, b_i into e_i, f_i and out of it.

3. Illustration of the simplest mechanical model of the light particle

We shall call the system consisting of a positive gravitational precharge α and a negative gravitational precharge $\alpha*$, connected with each other, the prolon. Let it is positioned at the centre of an elementary light particle. The system consisting of a positive electric precharge β and a negative electric precharge $\beta*$, connected with each other, will be called the elon. It will be positioned at the periphery of the elementary light particle. We shall call the elementary light particle the baron.

Let an elon moves mechanical around a prolon in a stationary orbit. We will consider four stages of such cyclic movement. They are presented in Fig.1.

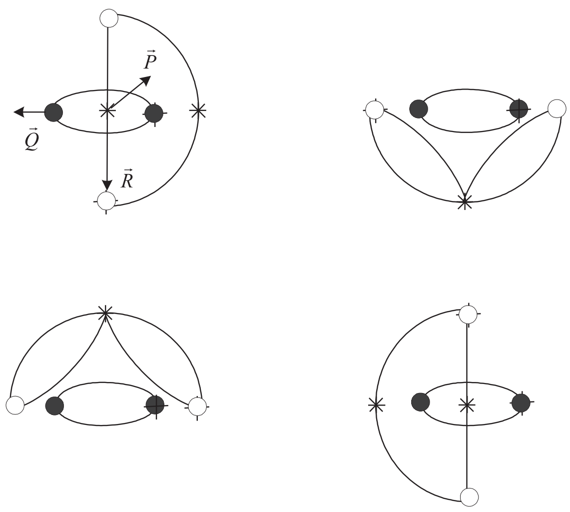

Fig. 1. Mechanical movement of the baron elements

We shall show, within the limits of the pattern of movements presented in Fig.1, that experimentally significant conclusions on the behaviour of light can be made without considering the law of the precharges interaction.

Let us introduce the vector $\vec{R}$ determining the direction from the negative electric precharge to the positive one in the baron. Let the vector $\vec{Q}$ determines the direction from the positive gravitational precharge to the negative one . Let us introduce the vector $\vec{P}$ perpendicular to $\vec{Q}$ and forming a right-hand system with it (Fig.1).

The fields $\vec{E}$ and $\vec{B}$ will be determined by the formulas $\vec{E} = a\vec{P}\left(\vec{R}\vec{Q}\right)$, $\vec{B} = b\vec{Q}\left(\vec{R}\vec{Q}\right)$, where $\left(\vec{R}\vec{Q}\right)-$ is the scalar product of the vectors. We shall obtain the known experimental result: an electromagnetic radiation is characterized by the values of $\vec{E}$ and $\vec{B}$, which change cyclically and consistently with each other and rich a maximum or a minimum at a time. Within the limits of the simplest mechanical model of a baron, this fact is explained by the cyclicity of movement of the electric precharges around the gravitational precharges.

4. Deduction of formulas for the Planck's constant and the energy of the light particle

We shall derive an expression for the energy of the simplest light particle, starting from the following model:

- the simplest light particle consists of an elon and a prolon, located, by analogy with an electron and a proton, in a hydrogen atom;
- an elon and a prolon represent nonpoint neutral objects consisting of positive and negative electric and gravitational precharges connected with each other by the receptors in the form of force tubes;
- a prolon represents a neutral analog of a proton, containing positive and negative premasses connected by the premass force tubes.
- an elon represents a neutral analog of an electron, containing positive and negative electric precharges connected by the preelectric force tubes.
- a prolon has a nonzero electric precharge, and an elon has a nonzero premass charge.

We shall consider a baron as a physical object consisting of an elon rotating around a prolon. It will be assumed that the receptors are real force lines as do precharges, formed from the oriented strings capable to form longitudinal and transverse connections. It should be noted that the physical medium, in which the elons and prolons are found, can has a complex composition and a complex structure.

We shall use the algorithm of analysis of the energy of the force tubes in the "light hydrogen", proposed by Thomson for electric charges [11] . He used the following formula for the energy E of a force tube:

$$\varepsilon_o E = 2\pi f^2 V.$$

Here $f-$ is a dielectric polarization, $V-$ is the volume of a force tube. A force tube connects a positive and negative electric precharges q . The outer radius of the ring of a force tube will be denoted by r and the radius of its cross-section will be denoted by b . The coefficient $p \leq 1$ accounts for the arrangement of the force lines in the force tube.

The polarization is calculated by the formula

$$f \cdot S = f \cdot \pi b^2 = p \cdot q.$$

For the energy of the force tube, simulating a light particle, we obtained the expression

$$E = 8\pi^2 \left(p\frac{r}{b}\right)^2 \frac{q^2}{\varepsilon_0 c\left(q\right)}\omega = \hbar\left(q\right)\omega.$$

The quantity

$$\hbar\left(q\right) = 8\pi^2 \left(p\frac{r}{b}\right)^2 \frac{q^2}{\varepsilon_0 c\left(q\right)},$$

as will be shown below, is an analog of the Plank constant for a precharge. Let us combine barons as the system in the form of a linear molecule consisting of N precharges connected with each other. Let $Nq = e$ is the electric charge of an electron: $e = 1.6021892 \cdot 10^{-19}$ Kl. Let, in this case, the peripheral velocity of movement of the precharges around the centre of the system is equal to the velocity of light in the vacuum $c\left(e\right) = 2.9979256 \cdot 10^8 m \cdot c^{-1}$. We obtain the standard equation

$$E = \hbar\omega$$

.

The calculated value of the Plank constant $\hbar$ is equal to the experimental one if

$$p\frac{r}{b} = 0.37226.$$

The frequency is determined from the formula

$$\omega = \frac{c}{2\pi \cdot r}.$$

It has a standard sense of the frequency of mechanical rotation of an elon around a prolon.

The standard quantum model of an electromagnetic field physically explains the discrete structure of light by the existence of the structureless light quanta. It phenomenologically uses the formula for an "energy portion" .

The structural model of light allows one to determine both the formulas of the light particle energy and the Planck constant. Discrete structure of the light new theory explains to the existence of the barons as elementary part of the light particle. A light particle can be formed from N elementary blocks, each of which rotates around the centre with a frequency ω. Let us assume that the energy connecting the blocks with each other is equal to zero. Then the energy of a light particle will be equal to the sum of the energies of its individual blocks. Therefore,

$$E = \hbar\omega = N\left(\frac{\hbar}{N}\right)\omega.$$

Consequently, the Planck constant of an individual block in a light particle consisting of N blocks is equal to

$$\hbar(1) = \frac{\hbar}{N}.$$

A large light object according to the quantum theory is composed of small objects according to the classical theory.

Conclusions

The generalized Maxwell equations for the moving media can be written in the matrix form on the basis of the filling group determining the relation between four physical objects. Since an electromagnetic field is electrically and gravitationally neutral, it may be suggested that an electromagnetic radiation structurally represents a system of physical particles, the properties of which should be determined theoretically and experimentally, which can be done with the use of simple structural models consistent with the experiment.

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